Review and takeaways from Lecture 7 Interpolation \& Pulse Shaping

- Discrete-time to continuous-time conversion requires interpolation
- Interpolation is a filtering operation
- It is convenient to use an FIR filter, but IIR could be used as well
- Interpolation filter has lowpass frequency selectivity
- Common pulse shapes:
- Infinite two-sided sinc (IIR)
- Truncated sinc (FIR)
- Rectangular (FIR)
- Triangular (FIR)
- Raised cosine (IIR)
- Truncated raised cosine (FIR)
- Pulse shape for discrete-time to continuous-time conversion (interpolation):
- Desired frequency range is $-\frac{1}{2} f_{s}<f<\frac{1}{2} f_{s}$
- Zero crossings at multiples of $T_{s}$ (except origin)
- Pulse shape for discrete-time to discrete-time conversion (interpolation):
- Desired frequency range is $-\frac{\pi}{L}<\omega<\frac{\pi}{L}$
- Zero crossings at multiples of $L$ (except origin)


## [10:50am - 11:05am] Announcements: Spectrum Regulation and Auctions

## http://users.ece.utexas.edu/~bevans/courses/realtime/lectures/13 Digital PAM/announcements.html

## [11:05am - 11:25am] Digital Pulse Amplitude Modulation (Slide 13-3)

Goal: convert bit stream to analog pulses.

- Serial-to-parallel conversion: group stream of bits into symbols
- Constellation maps symbol of bits to amplitude
- A symbol of $J$ bits requires $2^{J}$ levels in constellation
- The symbol of bits can be thought of as index into lookup table

| 4-PAM <br> Example | Symbol of Bits | Symbol Amplitude |
| :--- | :---: | :---: |
|  | 00 | $d$ |
|  | 01 | $3 d$ |
|  | 10 | $-d$ |
|  | 11 | $-3 d$ |

$M$-level PAM:

- $\quad M$ is the number of symbol amplitudes: $M=2^{J}$
- The symbols are separated by a period $T_{\text {sym }}=1 / f_{\text {sym }}$
- $\underbrace{\text { Bit Rate }}_{\text {bits/second }}=\underbrace{J}_{\text {bits/symbol }} \times \underbrace{f_{\text {sym }}}_{\text {symbols/second }}$

Binary phase shift keying (BPSK) is equivalent to 2-PAM
Q: What is the reasoning behind the ordering in the constellation map?
A: Different orderings are possible. An alternative example is:

| Two's complement | Symbol of Bits | Symbol Amplitude |
| :---: | :---: | :---: |
| 1 | 01 | $3 d$ |
| 0 | 00 | $d$ |
| -1 | 11 | $-d$ |
| -2 | 10 | $-3 d$ |

Q: Are the amplitude values in the constellation in analog units?
A: Yes, they represent amplitudes of the continuous-time analog waveform.

## [11:33am - 11:43am] BPSK/2-PAM Example (Slide 13-4)

Symbol amplitudes marked as circles. For 2-PAM, amplitudes are $-d$ and $+d$.

- $d=1$ Volt
- $T_{\text {sym }}=4 \mathrm{~ms}=\frac{100 \mathrm{~ms}}{25 \text { periods }}$

The maximum symbol amplitude is $(M-1) d=d$. The maximum amplitude in the baseband PAM signal is
 less than $2(M-1) d=2$

## [11:43am - 11:45am] 4-PAM Example (Slide 13-6)

Symbol amplitudes marked as circles. For 4-PAM, symbol amplitudes are $-3 d,-d,+d,+3 d$.

- $d=2$ Volt
- $T_{\text {sym }}=3 \mathrm{~ms}=\frac{60 \mathrm{~ms}}{20 \text { periods }}$

The maximum symbol amplitude is $(M-1) d$. The maximum amplitude in the baseband PAM signal is less
 than $2(M-1) d=6$
[11:40am - 11:50am] PAM Transmission (Slide 13-7)


## [11:50am - 11:55am] Digital Interpolation Example (Slide 13-8)

Example: increase sampling rate from 44.1 kHz to 176.4 kHz

- Upsample by 4
- Apply FIR lowpass filter to "fill in" inserted zeros

The input CD audio signal sampled at $f_{s_{1}}=44.1 \mathrm{kHz}$. The maximum frequency $f_{\text {max }}$ that could be captured is $f_{\text {max }}=\frac{1}{2} f_{s_{1}}=22.05 \mathrm{kHz}$.

Upsampling by $L$ increases the sampling rate by a factor of $L$, i.e. $f_{s}=L f_{s_{1}}$. Any frequency content at or above $f_{\text {max }}$ are artifacts introduced by upsampling. The interpolation filter should attenuate frequencies at or above $f_{\max }$ which corresponds to discrete-time freq.

$$
\omega_{\max }=2 \pi \frac{f_{\max }}{f_{s}}=2 \pi \frac{f_{\max }}{L f_{s_{1}}}=2 \pi \frac{f_{\max }}{2 L f_{\max }}=\frac{\pi}{L}
$$

